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SKEW QUADRUPOLE FOCUSING LATTICES AND APPLICATIONS*

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Abstract

In this paper we revisit using skew quadrupole fields in place of traditional normal upright quadrupole fields to make beam focusing structures. We illustrate by example skew lattice decoupling, dispersion suppression and chromatic correction using the neutrino factory Study-II muon storage ring design. Ongoing BNL investigation of flat coil magnet structures that allow building a very compact muon storage ring arc and other flat coil configurations that might bring significant magnet cost reduction to a VLHC motivate our study of skew focusing.

1 INTRODUCTION

In discussions of skew focusing lattices the author is sometimes confronted with the assertion that skew focusing automatically implies fully coupled lattices that are hard to deal with. We make the distinction, as others do [1], between betatron motion with two independently adjustable eigentunes, what we take for the meaning of decoupled motion, and simple horizontal and vertical (X,Y) betatron motion correlation, which is sometimes taken loosely as a signature of coupling. With pure skew focusing there can be completely independent betatron motion in each of two eigenplanes, denoted (A,B), but these eigenplanes are oriented at ±45° with respect to a conventional (X,Y) coordinate system. Betatron motion in such a lattice is most simply described via the (A,B) coordinates and results can be linearly transformed between (A,B) and (X,Y) according to the following rotation relations:

$$X=(A+B)/\sqrt{2}, \quad Y=(A-B)/\sqrt{2}$$

or

$$A=(X+Y)/\sqrt{2}, \quad B=(X-Y)/\sqrt{2}.$$

A simple realization of a skew focusing coil structure, powering two flat racetrack pancake coils in opposition, is shown schematically in Fig. 1. We see that a test particle that is offset along either diagonal eigenplane, $X=Y$ or $X=-Y$, gets a kick directed along the diagonal that keeps the particle oscillating in its original eigenplane. Such a coil configuration is attractive for muon storage ring superconducting magnets because the conductors are

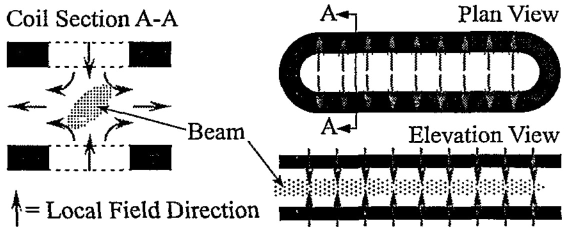

Figure 1: A simple pancake coil skew focusing structure.

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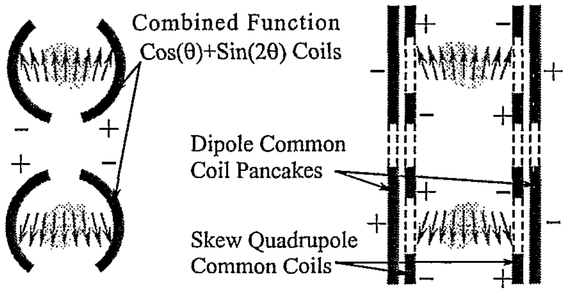

Figure 2: Two schemes for a double aperture VLHC magnet that incorporate skew quadrupole focusing.

kept away from the midplane where a large decay particle flux is found. If the top and bottom coils are powered with the same polarity a vertical dipole field results. Turning off either a top or bottom coil gives a combined function magnet with vertical dipole and skew quadrupole fields. Such flat coils are easily wound and can be overlapped in compact lattice structures that make efficient use of longitudinal space [2,3].

Two other skew focusing schemes, involving double aperture VLHC magnets, are illustrated in Fig. 2. On the left is a Cos(θ) inspired coil configuration, presented by Palmer [4], that attempts to reduce VLHC magnet costs by making longer magnet structures and having fewer coil ends. In this scheme the conductors move periodically up and down to provide both a vertical guide field and alternating F and D skew focusing fields.

The double aperture racetrack coil shown on the right in Fig. 2 follows Gupta's common coil VLHC magnet scheme [5] and avoids crossing conductors over beam pipes. The inner pancake coils generate skew quadrupole fields and when used alone give a separated function magnet. When the skew coils are combined side by side with dipole common coil layers the result is a combined function magnet. Such pancake skew focusing structures provide lattice focusing while maintaining the attractive simplicity of the common coil philosophy.

Clearly simple coil structures exist that naturally accommodate skew focusing via deliberate top-bottom coil asymmetries. We now show how such elements can be arranged to achieve various lattice design goals.

2 SKEW OPTICS DESIGN

2.1 Optics Decoupling

For a skew lattice an upright quadrupole field can mix the (A,B) eigenplane normal modes and thus act as a source of coupling. As observed previously by others [1], the weak normal focusing from sector dipole bends in a circular accelerator will cause coupling if left uncorrected; they deal with this issue by using dipoles with virtual field boundaries half way between sector and rectangular

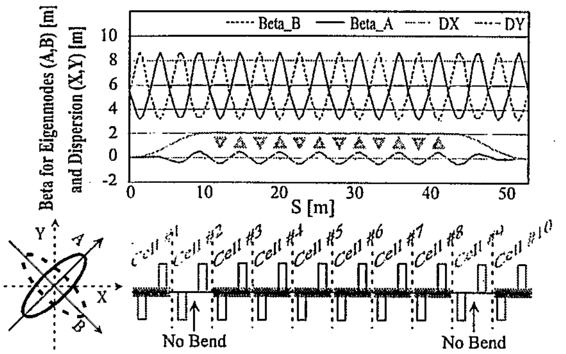

Figure 3: Eigenmode beta functions (A,B) and effective dispersion (X,Y) plotted for a muon storage ring arc with 60° cells. (A,B) and (X,Y) coordinate systems are shown. Zero bend cells suppress the dispersion in both planes.

bending. Their choice gives similar weak focusing in both planes and thus restores approximate azimuthal focusing symmetry; however, for a non-zero length dipole this compensation is not perfect due to the phase advance between the dipole ends.

Since we are interested in studying combined function magnets, with rather complicated field overlaps, we want to be able to arbitrarily subdivide lattice elements. But this becomes complicated with the above prescription as we are forced to give different edge rotations to internal subdivisions and true magnet ends to avoid having the optics results depend upon how elements are subdivided.

We find it simpler not to change the edge focusing but rather to ensure azimuthal symmetry by adding an upright quadrupole focusing component, with strength $k=-1/(2\rho^2)$, to every sector dipole of bend radius ρ. Such modified elements can be subdivided without affecting the optics and there is no coupling build up even with long sector bend magnets. This is not to say that weak sector focusing is thereby eliminated. It still adds to the net focusing, but when locally compensated, it does not bring about (A,B) eigenmode coupling.

We find this trick is also useful in a normal upright quadrupole focusing lattice. For example a simple FODO cell, with equal F and D focusing strengths, will still have slightly different (X,Y) beta functions and phase advances due to sector bend weak focusing. With the above trick,

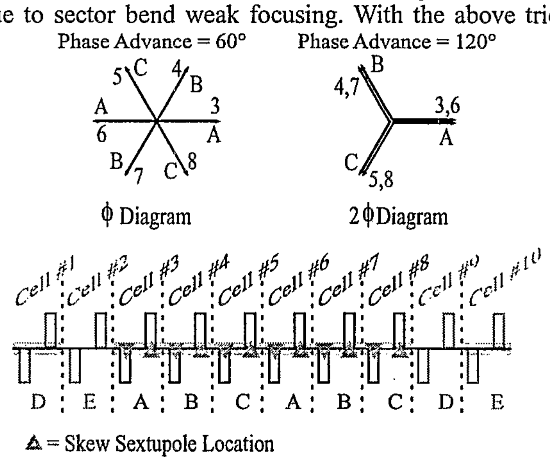

Figure 4: Skew sextupole chromaticity correction scheme.

adding the extra focusing in the sector bends, the optics become precisely the same in X and Y.

By extension we develop a simple prescription for converting a decoupled upright focusing lattice into a decoupled skew lattice as follows. First design the upright lattice with $-1/(2\rho^2)$ normal focusing added to all sector bends. Then the main quadrupole fields can be rotated 45° to create the desired skew focusing lattice. The skew lattice remains decoupled due to local azimuthal symmetry in the modified sector bends.

In many optics design codes it is a nontrivial job to obtain the (A,B) eigenplane optics when skew quadrupoles are the main focusing elements. For this we offer a second related trick: do linear optics calculations with the dipoles rotated by $-45°$ and keep the main quadrupole fields upright. With the identifications, $B\equiv X$ and $A\equiv Y$, lattice functions, tunes etc. for the (A,B) eigenmodes can be read directly from standard output (without invoking special modules for computing coupled mode optics).

Adding a small amount of normal focusing is quite straightforward with our proposed muon storage ring dipole racetrack coil design. A small normal component is introduced by having slightly different coil separations on each side of the beam (e.g. tipping the top and bottom pancake coils in opposite directions). For the Study-II dipole design this coil position adjustment is smaller than 1 mm, compared to the 130 mm nominal coil separation, and can easily be accommodated [2,3].

2.2 Dispersion Suppression

Dispersion suppression with skew focusing lattices goes much the same as normal. For a FODO arc with phase advance in each plane μ per cell, we adjust the bending in the two cells at each end to be the fractions x, and $1-x$ of a standard cell. With $x=1/[2(1-Cos(\mu))]$ the arc dispersion is matched to zero outside. For $\mu=90°$, we have $x=1-x=1/2$. We tested this simple prescription for 90° phase advance and it works well both with separated function and combined function lattices.

A second example, our muon storage ring design with $\mu=60°$, is shown in Fig. 3. Here $x=1$ and $1-x=0$ so we must remove all bending from the next to last cells at each end of an arc. The (A,B) arc beta functions and dispersion are equal (up to a phase). We use the above rotation relations to transform to (X,Y) and find that the eigenmodes add constructively for X to give an almost constant horizontal dispersion and destructively for Y to leave a small oscillatory vertical dispersion.

We note an important technical point here; because the empty cells have modified bending they do not have the same weak sector focusing as found in an arc cell. The quadrupole strengths and positions in empty cells have to be adjusted to match the arc cell lattice functions and 60° phase advance. Our muon storage ring arc cells use superconducting magnets and the empty cells normal conducting quadrupoles; we can easily set up a modified empty cell that matches the standard arc cell (the empty cells also provides warm space for beam collimation) [3].

2.3 Chromaticity Correction

For chromatic correction in the muon storage ring we place a skew sextupole at each F and D skew quadrupole in the central section of the arcs as shown in Fig. 3 and Fig. 4. Because 45° is not a multiple of 30° we choose to use skew sextupoles for chromaticity correction in place of normal sextupoles since this choice maintains the same relative orientation between the skew eigenplanes and the skew sextupole as is found between the eigenplanes and normal sextupoles in an upright lattice. We did experiment with normal sextupoles to correct skew lattices but found that much larger strengths were required than with skew sextupoles. This may explain the large sextupole strengths reported by others [1]. Also since skew sextupole magnets have no midplane conductors they are compatible with the muon storage ring coil structures described earlier.

For the 60° arc lattice we use three skew sextupole families, **A**, **B** and **C**, that are arranged so there is automatic first-order and higher-order cancellation between the nonlinear terms as illustrated in Fig. 4. With one degree of freedom needed to correct chromaticity and another needed to correct second-order nonlinearities, there is still one remaining degree of freedom in each plane available to minimize the nonlinear momentum dispersion in the neutrino production straight [3].

We also investigated alternate arc designs with 90° cells for both separated function and combined function lattices and as expected were able to reproduce standard upright lattice results using just two skew sextupole families.

2.4 Synchrotron Radiation Damping

One desirable dynamical effect associated with synchrotron radiation in a high field VLHC is transverse emittance damping. For a skew focusing VLHC, designed with equal arc *(A,B)* eigenplane beta functions and dispersions, we apply results from [1], and expect to see equal damping in both planes. Equilibrium "round beams" with equal emittance in both eigenplanes are thus predicted; however, unequal emittance flat beams might be desired to allow doublet interaction region optics.

To see that it is possible to make flat beams with skew lattices we consider a toy VLHC lattice that has different cell phase advance in the two eigenplanes (specifically 60° and 90°). By removing dipoles at appropriate phase differences it is possible (e.g. since 180° phase differences occur at different locations in the two eigenplanes) to arrange for the average value of $H = \eta^2 + 2\alpha\eta\eta' + \eta'^2$ in the arc dipoles to be small in one eigenplane at the cost of increasing it in the other eigenplane. Thus we demonstrate one way to make a flat beam with a skew lattice, shifting damping between the two eigenplanes; undoubtedly there are other less drastic ways that should be explored if flat beams really are needed for the VLHC.

2.5 Aperture Requirements and BPM Design

Two undesirable effects associated with synchrotron radiation in a high field VLHC are heating of the beam pipe and gas desorption. Their cure typically involves

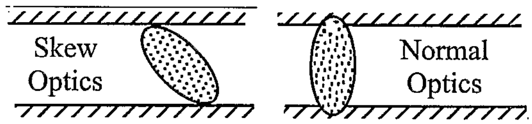

Figure 5: Skew and normal optics aperture comparison with same vertical acceptance cut, beta functions and emittances. For skew acceptance $\beta_{eff} = (\beta_A + \beta_B)/2$.

introduction of a secondary beam screen that can take up vertical aperture as indicated schematically in Fig. 5. Beam circulating in our muon storage ring magnet also has to pass through a non-circular aperture. For both cases the imposition of a non-circular cut on beam aperture can give a small advantage to a skew optics solution. For skew optics the projected beam size goes as square root of the effective beta, $\beta_{eff} = (\beta_A + \beta_B)/2$, rather than square root maximum beta. For a circular aperture or at a location where the eigenplane beta functions are equal there is no special skew optics benefit but otherwise we are able to accept larger beams by taking advantage of the diagonal.

We also expect that the synchrotron radiation fan generated with skew focusing optics should be fairly constant in vertical size and thus not have the same hot spots that are possible with upright focusing optics.

Since the fundamental skew optics eigenmodes are aligned along diagonals, it makes sense to put BPM pickups out of plane along the diagonals. Then we can look for a favorable geometry with skew optics where the synchrotron radiation fan and decay particles do not automatically strike the beam pickups.

3 SUMMARY

Traditionally lattice designers have not made use of skew quadrupole fields for main focusing and some optics codes have built in assumptions that can make it difficult to do skew optics calculations; however, we see that there are magnet coil structures and situations where skew focusing may prove useful. The examples given here were taken from recent muon storage ring and VLHC design work. For these cases we found that many standard optics procedures still work fine and pointed out a few simple tricks and useful relations. There appears to be a natural correspondence between upright and skew focusing lattices that we encourage the reader to explore further.

4 REFERENCES

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